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THEORY ON TRANSVERSE MIGRATION OF PARTICLES IN A TURBULENT TWO-PHASE SUSPENSION FLOW DUE TO TURBULENT DIFFUSION—I

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Abstract—A new theoretical model has been developed to explain the behavior of transverse particle transport in turbulent flow of a dilute two-phase suspension due to turbulent diffusion. This model is based on the ability of a particle to respond to surrounding fluid motion and depends on particle size and density relative to the carrier fluid, the fractional variation in particle concentration in the transverse direction as well as the existing turbulence structure of the surrounding fluid. The model developed in this investigation has been formulated by dividing the transverse fluid velocity, as seen by a particular particle, into two superimposed components representing, respectively, the transverse turbulent fluid fluctuations and an apparent transverse local fluid drifting velocity due to the effect on the transverse oscillatory component of fluid motion by the transverse concentration distribution of particles. A subsequent paper will show that the theory (together with other new results on the concentration effects on particle drag and lift and fluid turbulence properties) can help to explain the phenomena measured previously.

INTRODUCTION

A comprehensive understanding of transverse particle motion in a turbulent dilute two-phase suspension flow due to turbulent diffusion is of fundamental importance. A model that successfully analyzes the physics behind particle transport can easily lend itself to explaining many problems currently being researched in this field. Particle deposition rates, for one, cannot be accurately solved without a basic understanding of this important flow mechanism which contributes to transporting the particles towards the wall from the flow. This investigation, therefore, addresses and attempts to describe the mechanism behind such a two-phase turbulent flow.

Most conventional theoretical treatments of particle transport flow fields assume that two distinct flow regimes exist: the viscous sublayer and the turbulent diffusion core region, established from knowledge obtained from single-phase flows (e.g. Lin *et al.* 1953; Friedlander & Johnstone 1951; Davies 1966; Beal 1968; Hutchinson *et al.* 1971). In the fully developed turbulent core region, particles are assumed to be laterally transported by turbulent diffusion in the same way as scalar quantities are assumed to be transported in a turbulent stream. Particles reaching the edge of the viscous sublayer as a result of this transport are assumed to coast towards the wall across the sublayer to form deposition. Unfortunately, these conventional treatments all contain an adjustable constant which is not universal for all flow systems. For instance, Wildi (1982) compared the prediction from one such treatment with the result of wall-deposition measurement of a mist flow of droplets over a size range. With the empirical constant adjusted to produce reasonable prediction for the larger droplets, the treatment generates an awkward underprediction of 4 orders of magnitude for the smaller droplets. This apparent drawback points to the question of the correctness of the very physics assumed in these theoretical treatments.

Rouhiainen & Stachiewicz (1970) pointed out that the particles' response to turbulent fluid motion is important for particle wall deposition. Their examination leads to the realization that the particle turbulent diffusivity, ϵ_p, equals the fluid turbulent diffusivity, ϵ_f, only for very small particles. The theoretical treatment by them, first developed by Hjelmfelt & Mockros (1966), is based on the concept of particle frequency response in an oscillating flow field. An important consequence of this approach is that the validity of the turbulent diffusion assumption for particle transport in a turbulent fluid stream can be characterized by the value of an amplitude ratio, η,

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which is defined as the ratio of the amplitude of particle oscillation to that of the surrounding eddy motion. The main weakness of Rouhiainen & Stachiewicz's (1970) analysis lies in its lack of suggestion of a rational scheme to link the two distinctly different flow regions, the turbulent diffusion core and the laminar or quasi-laminar viscous layer.

Lee & Durst (1979, 1980, 1982) have suggested a new general theoretical approach to transverse particle transport in turbulent flow. Their approach is based on particles' dynamic response characteristics to the surrounding eddy motion. Their analysis divides the flow field into two transverse regions: the turbulent diffusion controlled core region where particle motion is controlled by turbulent fluid oscillations, so that $\epsilon_p = \epsilon_f$; and the mean fluid motion controlled quasi-laminar region where particle motion is controlled by mean fluid motion so that $\epsilon_p = 0$. The boundary between these two regions is determined by the cutoff radius r_c, which according to the analysis is a function of particle size, flow properties and physical system properties. The cutoff radius was defined as the radial distance from the pipe centerline to the point where the amplitude ratio, η, as devised by Rouhiainen & Stachiewicz (1970), equals one-half. Some of their theoretical conclusions were qualitatively verified by local velocity measurements performed by laser–Doppler anemometry in particulate two-phase flows.

The drawback of Lee & Durst's (1979, 1980, 1982) approach, however, arises because an artificial boundary was placed between the two flow regions; i.e. it is not possible to have a smooth transition of flow properties in the transverse direction. Most seiously this model treats the two regions separated by this artificial boundary as separate unrelated identities. Consequently, the analysis contains two unconnected sets of governing equations having no physical or mathematical ties with each other. In addition, the particle turbulent eddy diffusivity, ϵ_p, which controls the transverse particle transport in the turbulent core, has been assumed to be constant and equal to the turbulent eddy diffusivity of the surrounding fluid, ϵ_f.

The model now being proposed is directed towards an extension of Lee & Durst's (1979, 1980, 1982) model. One single set of governing equations then should dictate the particles' response to the turbulent fluid motion. As intuitively expected, a smooth transition in flow properties should occur in traversing the pipe. Strict notation of the flow region is no longer necessary since the flow field is governed by one set of equations. In particular, the turbulent diffusion of particles should be effective in varying degrees across the whole pipe. Therefore, it is necessary to come up with a model to relate the local particle turbulent eddy diffusivity, ϵ_p, to the local fluid-flow turbulence structure.

The analysis begins with a discussion of the governing equation of motion of a spherical particle in a moving turbulent flow. Basset (1888) originally derived these equations for the slow motion of a particle in a stationary fluid. Tchen (1947) modified them for application to a moving turbulent flow. This is followed by a development of the proposed model. The analysis is then broken down into two sections corresponding to the two components of fluid velocity, that when superimposed onto one another comprise the fluid velocity in the transverse direction as seen by a particular particle. As the analysis points out, two governing equations for particle motion involving each of the two fluid velocities separately, can be extracted from Tchen's (1947) single governing equation. These two equations, their solutions, associated simplifications and physical representations are discussed and analyzed. It is this set of differential equations that describe the basic physics with which the particle frequency response can be explained.

THEORETICAL MODEL

The governing equation of motion of a spherical particle in a turbulent fluid flow, first formulated by Tchen (1947) and reviewed by Hinze (1959) and subsequently rederived from first principles by Maxey & Riley (1983), can be written as follows:

$$\frac{\pi}{6} d_p^3 \rho_p \frac{d\mathbf{V}_p}{dt} = 3\pi\mu_f d_p(\mathbf{V}_f - \mathbf{V}_p) + \frac{\pi}{6} d_p^3 \rho_f \frac{d\mathbf{V}_f}{dt} + \frac{1}{2}\frac{\pi}{6} d_p^3 \rho_f\left(\frac{d\mathbf{V}_f}{dt} - \frac{d\mathbf{V}_p}{dt}\right) + \tfrac{3}{2} d_p^2(\pi\rho_f\mu_f)^{\frac{1}{2}} \int_0^t \frac{\left(\frac{d\mathbf{V}_f}{dt'} - \frac{d\mathbf{V}_p}{dt'}\right)}{(t-t')^{\frac{1}{2}}} dt', \quad [1]$$

where d_p is the particle diameter, ρ_p and ρ_f are the intrinsic particle and fluid densities, $\mathbf{V}_p$ and $\mathbf{V}_f$ are the particle and fluid velocities, and μ_f is the molecular viscosity of the fluid.

The sum of the terms on the r.h.s. of this equation is equal to the force required to accelerate a particular particle in the flow field, the first term on the r.h.s. is the viscous force in the form of Stokes' drag; the second term is the force generated by the pressure gradient in the fluid surrounding the particle due to fluid acceleration; the third term is the force needed to accelerate the essential "added" mass of the particle; the final term, known as the Basset history term, accounts for steady-state flow deviations.

Hjelmfelt & Mockros (1966) point out that there are four requirements related to conventional studies incorporating [1]. Except for one modification, they all apply to this investigation. First, the turbulence must be locally homogeneous and stationary. Second, the particle diameter, d_p, must be substantially less than the length scale of the energy-containing eddies. This requirement pertains only to particles within the particle transport region of the turbulent diffusion controlled range for the present model. Third, a small Reynolds number based on the relative velocity of the fluid and particle must exist. Lastly, particle concentrations are considered dilute enough such that the presence of the particles does not interfere with the overall known fluid motion.

In addition, [1] is subject to the following restrictions, as pointed out by Corrsin & Lumley (1956) and discussed by Hinze (1959):

$$\frac{d_p^2}{\nu_f}\frac{\partial \mathbf{V}_f}{\partial x} \ll 1, \quad \frac{\mathbf{V}_f}{d_p^2\left(\dfrac{\partial^2 \mathbf{V}_f}{\partial x^2}\right)} \gg 1, \tag{2}$$

where ν_f is the intrinsic kinematic fluid viscosity. These restrictions are a direct result of the Navier–Stokes equation for a single component of velocity:

$$-\frac{\partial p}{\partial x_i} = \rho_f\left[\frac{\partial}{\partial t}(\mathbf{V}_f)_i + (\mathbf{V}_f)_k\frac{\partial}{\partial x_k}(\mathbf{V}_f)_i\right] - \mu_f\frac{\partial^2(\mathbf{V}_f)_i}{\partial x_j\,\partial x_j}. \tag{3}$$

It should be noted that the local pressure gradient surrounding a particle is due to viscous effects as well as fluid accelerations. Tchen's (1947) analysis only considers fluid accelerations. Hinze (1959) points out that because the particle diameter is much smaller than the associated turbulent length scale, and the time intervals are shorter than associated deformation times, the fluid velocity may be considered uniform and the viscous effects associated with the pressure gradient considered negligible. The restrictions placed on [1] are therefore met. Written for the transverse direction, [1] becomes

$$\frac{\mathrm{d}v_p}{\mathrm{d}t} + \mathrm{a}v_p = \mathrm{a}v_f + \mathrm{b}\frac{\mathrm{d}v_f}{\mathrm{d}t} + \mathrm{c}\int_0^t \frac{\left(\dfrac{\mathrm{d}v_f}{\mathrm{d}t'} - \dfrac{\mathrm{d}v_p}{\mathrm{d}t'}\right)}{(t-t')^{\frac{1}{2}}}\,\mathrm{d}t', \tag{4}$$

where

v_p, v_f = particle and fluid velocity components in the transverse direction, respectively,

$$\mathrm{a} = \frac{36\,\mu_f}{(2\rho_p + \rho_f)d_p^2}, \tag{5a}$$

$$\mathrm{b} = \frac{3\rho_f}{(2\rho_p + \rho_f)} \tag{5b}$$

$$\mathrm{c} = \frac{18}{(2\rho_p + \rho_f)d_p}\left(\frac{\rho_f\mu_f}{\pi}\right)^{\frac{1}{2}}. \tag{5c}$$

Actual particle transport flows contain particles and eddies of various sizes. For simplicity, [1] considers only spherical particles of one specific diameter, d_p. It is to be pointed out that the model presently being developed can be readily extended to incorporate a dispersion of particles of multiple sizes. A solid particle–gaseous fluid mixture has been chosen for this model so that the density ratio of particle/fluid considered is relatively large. Hinze (1959) points out that when this density ratio is large the second, third and fourth terms of [1] are of only slight consequence. The present analysis will therefore neglect the Basset history term cited as a type I approximation by Hjelmfelt & Mockros (1966), and therefore $c = 0$ in [4].

A type II approximation, according to Hjelmfelt & Mockros (1966), neglects the "added" mass and the Basset history terms, and a type III approximation neglects the pressure gradient, "added" mass and Basset history terms.

This analysis proposes that the local velocity of an incremental fluid volume as seen by a particle in a turbulent two-phase dispersion flow can be expressed as a summation of two velocities, the turbulent fluctuating velocity and a drifting or turbulent diffusion velocity due to the interaction between the turbulent fluctuating velocity and the transverse particle fractional concentration gradient.

Tchen (1947) was the first to suggest that turbulent fluid fluctuations may be expressed as Fourier integrals of the following form:

$$\tilde{v}_{fs} = \int_0^\infty (\xi \cos \omega t + \lambda \sin \omega t)\, d\omega, \qquad [6]$$

where ξ and λ are complex Fourier integral amplitudes that can be defined as

$$\xi(\omega) = \frac{2}{\pi} \int_0^\infty \tilde{v}_{fs}(t)(\cos \omega t)\, dt$$

and

$$\lambda(\omega) = \frac{2}{\pi} \int_0^\infty \tilde{v}_{fs}(t)(\sin \omega t)\, dt; \qquad [7]$$

ω is the frequency, equal to $2\pi n$. Stated differently, fluid fluctuations may be physically represented as having sinusoidal components.

Particles being transported in a turbulent flow, in general, will not be evenly distributed in the transverse direction and thus a local transverse particle concentration gradient usually exists. The fluctuating motion of the fluid will then be affected by this uneven distribution of particles. The fluid will find it more difficult to pass through in the direction of increasing particle concentration than in the direction of decreasing particle concentration. A larger drag will have to be overcome in the direction of increasing particle concentration. As a result of this uneven force distribution, the fluid oscillations will have uneven amplitudes, as shown in figure 1. This locally unbalanced sinusoidal motion leads to an apparent fluid drifting velocity away from the high particle

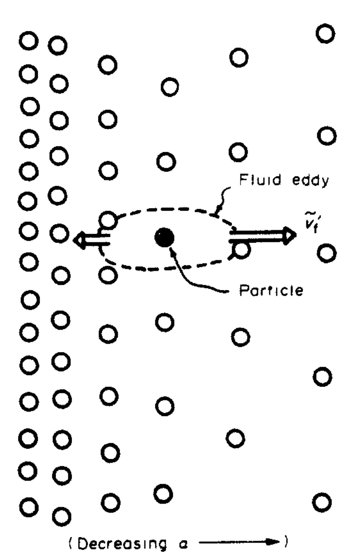

Figure 1. Uneven fluid eddy flow oscillation about a particle due to particle concentration variation.

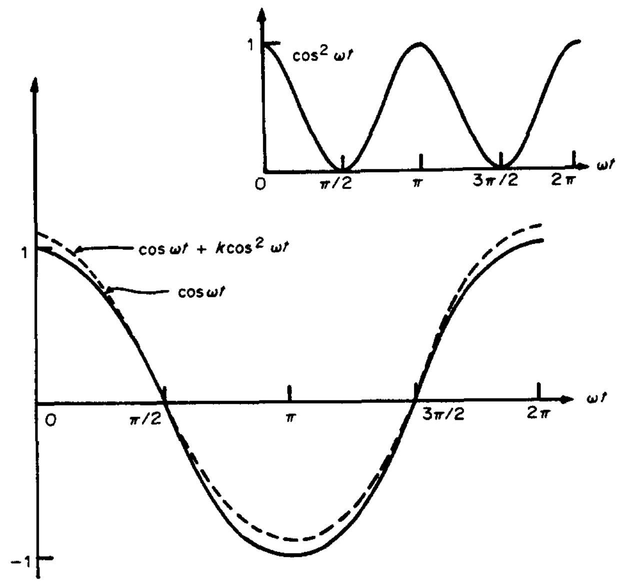

Figure 2. Schematic representation of the $k\cos^2\omega t$ component of apparent local fluid motion superimposed onto the $\cos\omega t$ component of fluctuating fluid motion.

concentration region. From a local point of view, on top of sinusoidal motion the fluid appears to be drifting in the direction of decreasing particle concentration.

This unbalanced sinusoidal motion can be obtained by perturbing the already established fluid velocity in such a way as to slowly shift the entire oscillating motion in one direction. The desired drifting effect is achieved by considering a trial generating function for the fluid velocity as follows:

$$\tilde{v}_f' = \int_0^\infty [\xi \cos \omega t(1 + k \cos \omega t) + \lambda \sin \omega t(1 + k \sin \omega t)]\, d\omega. \qquad [8]$$

A graphic description of the $[\cos\omega t(1 + k \cos\omega t)]$ component of this proposed fluid velocity is shown in figure 2. A qualitatively similar description of the $[\sin\omega t(1 + k \sin\omega t)]$ component of this proposed fluid velocity can also be shown. Rearranging the above expression, we have

$$\tilde{v}_f' = \tilde{v}_{fs} + \tilde{v}_{fd}, \qquad [9]$$

where

$$\tilde{v}_{fs} = \int_0^\infty (\xi \cos \omega t + \lambda \sin \omega t)\, d\omega$$

and

$$\tilde{v}_{fd} = \int_0^\infty [k(\xi \cos^2 \omega t + \lambda \sin^2 \omega t)]\, d\omega. \qquad [10]$$

This trial apparent transverse fluid velocity function, $\tilde{v}_f'$, is therefore comprised of two parts: a sinusoidal fluid motion expressed as a Fourier integral, $\tilde{v}_{fs}$; and an additional fluid motion due to the interaction between turbulent fluctuations and transverse fractional variations of particle concentration, $\tilde{v}_{fd}$.

Equation [9] points out that from a local point of view, the turbulent fluid motion can be divided into two components. The two terms $\sin^2\omega t$ and $\cos^2\omega t$ are always positive and have magnitudes $\leqslant 1$; therefore, as long as k is a small quantity, the apparent fluid velocity, $\tilde{v}_f'$, will start to shift in a direction according to the sign of k, as well as continue to oscillate. The physical reasoning behind this apparent drifting lends itself to say that the quantity k should be proportional to the local gradient of fractional particle concentration. In this light it will be assumed that

$$k = k' \frac{1}{\alpha} \frac{\partial \alpha}{\partial y}, \qquad [11]$$

where k' is a proportionality constant, to be determined, and α is the time-mean particle volumetric concentration.

Fluctuating quantities in turbulence are often separated into two components: a time-mean quantity; and a time-dependent quantity, which fluctuates around the mean quantity. Following this convention, we establish the following expression for the apparent fluid velocity corresponding to the unbalanced fluctuating motion:

$$v_f = \tilde{v}_{fs} + \bar{\bar{v}}_{fd}, \qquad [12]$$

where

$$\bar{\bar{v}}_{fd} = \int_0^\infty \int_0^{\frac{2\pi}{\omega}} \frac{k(\xi \cos^2 \omega t + \lambda \sin^2 \omega t)}{\left(\frac{2\pi}{\omega}\right)} \mathrm{d}t \, \mathrm{d}\omega \qquad [13]$$

is the apparent fluid drifting velocity based on [10] and $\tilde{v}_{fs}$ is the time-dependent sinusoidal fluid oscillatory velocity of [10].

It will be assumed that particle velocity counterparts exist in response to the two fluid velocities described above. Using the same notation,

$$v_p = \tilde{v}_{ps} + \bar{\bar{v}}_{pd}. \qquad [14]$$

It will also be assumed that each of the two particle velocities individually respond to each of the two fluid velocities. In other words, this investigation assumes that the sinusoidal particle velocity, $\tilde{v}_{ps}$, responds only to the sinusoidal fluid velocity, $\tilde{v}_{fs}$. Likewise, the particle drifting velocity, $\bar{\bar{v}}_{pd}$, is assumed to respond to the apparent fluid drifting velocity, $\bar{\bar{v}}_{fd}$. The two fluid velocities are separately treated as the drivers of two different fluid-particle velocity pairs.

Substituting [12] and [14] in [4], we have the two governing equations, respectively, for the two particle transverse velocity components $\bar{\bar{v}}_{pd}$ and $\bar{\bar{v}}_{ps}$ as follows:

$$\frac{\mathrm{d}\bar{\bar{v}}_{pd}}{\mathrm{d}t} + \mathrm{a}\bar{\bar{v}}_{pd} = \mathrm{a}\bar{\bar{v}}_{fd} + \mathrm{b}\frac{\mathrm{d}\bar{\bar{v}}_{fd}}{\mathrm{d}t} + \mathrm{c}\int_0^t \frac{\left(\frac{\mathrm{d}\bar{\bar{v}}_{fd}}{\mathrm{d}t'} - \frac{\mathrm{d}\bar{\bar{v}}_{pd}}{\mathrm{d}t'}\right)}{(t-t')^{\frac{1}{2}}} \mathrm{d}t' \qquad [15]$$

and

$$\frac{\mathrm{d}\tilde{v}_{ps}}{\mathrm{d}t} - \mathrm{a}\tilde{v}_{ps} = \mathrm{a}\tilde{v}_{fs} + \mathrm{b}\frac{\mathrm{d}\tilde{v}_{fs}}{\mathrm{d}t} + \mathrm{c}\int_0^t \frac{\left(\frac{\mathrm{d}\tilde{v}_{fs}}{\mathrm{d}t'} - \frac{\mathrm{d}\tilde{v}_{ps}}{\mathrm{d}t'}\right)}{(t-t')^{\frac{1}{2}}} \mathrm{d}t'. \qquad [16]$$

ANALYSES

Determination of particle fluctuating velocity due to fluid oscillations in the transverse direction, $\tilde{v}_{ps}$

The first type of fluid-particle motion to be discussed is the fluctuating velocity pair, $\tilde{v}_{fs}$ and $\tilde{v}_{ps}$. Hinze (1959) has contributed a great deal to the development of this particular particle frequency response. Hinze's equation of motion in terms of $\tilde{v}_{ps}$ and $\tilde{v}_{fs}$ is identical to [16]:

$$\frac{\mathrm{d}\tilde{v}_{ps}}{\mathrm{d}t} + \mathrm{a}\tilde{v}_{ps} = \mathrm{a}\tilde{v}_{fs} + \mathrm{b}\frac{\mathrm{d}\tilde{v}_{fs}}{\mathrm{d}t} + \mathrm{c}\int_0^t \frac{\left(\frac{\mathrm{d}\tilde{v}_{fs}}{\mathrm{d}t'} - \frac{\mathrm{d}\tilde{v}_{ps}}{\mathrm{d}t'}\right)}{(t-t')^{\frac{1}{2}}} \mathrm{d}t'. \qquad [16']$$

Keeping in mind that the fluctuating fluid velocity, $\tilde{v}_{fs}$, is assumed to be of the form

$$\tilde{v}_{fs} = \int_0^\infty (\xi \cos \omega t + \lambda \sin \omega t) \, \mathrm{d}\omega, \qquad [17a]$$

Hinze postulated that the resulting particle velocity is of the form

$$\tilde{v}_{ps} = \int_0^\infty \eta[\xi \cos(\omega t + \beta) + \lambda \sin(\omega t + \beta)] \, \mathrm{d}\omega. \qquad [17b]$$

This can be physically interpreted as saying a particle responding to fluid oscillations will in general be out of phase by the amount β, and have its amplitude modified by the factor η. In solving this differential equation Hjelmfeit & Mockros (1966) have obtained the following expression for the amplitude ratio η:

$$\eta = [(1+f_1)^2 + f_2^2]^{\frac{1}{2}}; \qquad [18]$$

and for the phase angle,

$$\beta = \tan^{-1}\left(\frac{f_2}{1+f_1}\right) \qquad [19]$$

where

$$f_1 = \frac{\omega\left[\omega + c\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right](b-1)}{\left[a + c\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2 + \left[\omega + c\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2} \qquad [20a]$$

and

$$f_2 = \frac{\omega\left[a + c\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right](b-1)}{\left[a + c\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2 + \left[\omega + c\left(\frac{\pi\omega}{2}\right)^{\frac{1}{2}}\right]^2}. \qquad [20b]$$

The amplitude ratio, η, and phase angle, β, are now seen to be functions of the fluid oscillation frequency, ω, and the intrinsic properties of both the fluid and the particle.

The amplitude ratio, η, has been shown by Hinze (1959) to be related to both the particle and eddy turbulent diffusivities, ϵ_p and ϵ_f, respectively, by

$$\frac{\epsilon_p}{\epsilon_f} = \frac{\int_0^\infty \eta^2 E_f(n)\,dn}{\int_0^\infty E_f(n)\,dn}, \qquad [21]$$

where $E_f(n)$ is the Lagrangian energy spectrum as a function of n, where $n = \omega/2\pi$, the frequency.

Therefore, in order to relate the fluid turbulent diffusivity, ϵ_f, to the particle turbulent diffusivity, ϵ_p, the Lagrangian description of the energy spectrum in the sense of following the fluid element must be employed. Experimental data, however, is almost exclusively collected using a Eulerian or control volume approach. Lee & Durst (1982) point out that because in most cases of practical flows of interest, such as flow through a straight pipe, the fluid velocity in the transverse direction is negligible on a gross scale, the Eulerian description may usually be used to approximate the Lagrangian description for transverse fluid motion.

When $\eta = 1$ in [21], the particle turbulent diffusion equals the fluid turbulent diffusion, and the particle's motion is completely controlled by fluid oscillations. When $\eta = 0$, particle turbulent diffusivity equals zero, and the particle's motion is completely governed by the mean fluid quasi-laminar (or laminar) viscous interaction of the mean motion in the surrounding flow field. A typical curve of the amplitude ratio, η, as a function of oscillating fluid frequency, constructed from [18] is shown in figure 3. Just as the time-mean average of the turbulent fluid oscillations can be shown to equal zero, the time-mean average of the oscillating particle velocity can be shown to equal zero.

It can be shown that the particle turbulent velocity in the transverse direction can be obtained as follows:

$$[\overline{(\tilde{v}_{ps})^2}]^{\frac{1}{2}} = \eta_e[\overline{(\tilde{v}_{fs})^2}]^{\frac{1}{2}}, \qquad [22]$$

where η_e is the amplitude ratio corresponding to ω_e; from [8] and [20],

$$\eta_e = \eta(\omega_e) = \{[1+f_1(\omega_e)]^2 + [f_2(\omega_e)]^2\}^{\frac{1}{2}} \qquad [23]$$

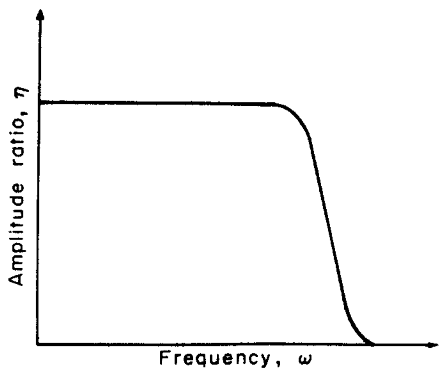

Figure 3. Particle frequency response.

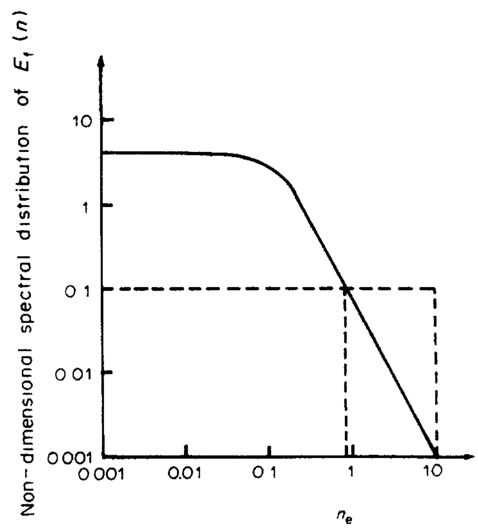

Figure 4. Typical spectral distribution of $E_f(n)$.

and ω_e, the frequency associated with the average turbulent kinetic energy of the fluid across the spectrum, can be provided from a typical energy spectrum curve as shown in figure 4.

Determination of particle drifting or diffusion velocity in the transverse direction, $\bar{\bar{v}}_{pd}$

We have from [13], after some manipulation, the apparent drifting velocity of the fluid due to the effect on the oscillatory component of fluid motion by the concentration distribution of particles:

$$\bar{\bar{v}}_{fd} = \frac{k}{2}\int_0^\infty (\xi + \lambda)\,d\omega. \tag{24}$$

For the trivial case of simple diffusion,

$$\frac{d\bar{\bar{v}}_{fd}}{dt} = 0 \quad \text{and} \quad \frac{d\bar{\bar{v}}_{pd}}{dt} = 0,$$

we have from [15], [24] and [11]:

$$\bar{\bar{v}}_{pd} = \bar{\bar{v}}_{fd} = \frac{k'}{2}\frac{1}{\alpha}\frac{\partial\alpha}{\partial y}\int_0^\infty (\xi + \lambda)\,d\omega. \tag{25}$$

Lee & Durst (1980) have proposed that the turbulent diffusion equation for particles in a suspension may be written as

$$\bar{\bar{v}}_{pd} = -\epsilon_p \frac{1}{\alpha}\frac{\partial\alpha}{\partial y}. \tag{26}$$

Comparing [25] and [26] produces an expression for the constant of proportionality, k':

$$k' = \frac{-2\epsilon_p}{\displaystyle\int_0^\infty (\xi + \lambda)\,d\omega}. \tag{27}$$

Substituting this expression for k' into [11] yields

$$k = \frac{-2\epsilon_p}{\displaystyle\int_0^\infty (\xi + \lambda)\,d\omega}\frac{1}{\alpha}\frac{\partial\alpha}{\partial y}. \tag{28}$$

It is desirable to express k, and hence $\bar{\bar{v}}_{fd}$ in quantities that can be obtained experimentally. Hinze (1959) demonstrated that

$$\int_0^\infty E_f(n)\,dn = \frac{1}{2}\int_0^\infty \frac{(\xi^2 + \lambda^2)}{T}\,d\omega = \overline{(\bar{v}_{fs})^2}, \tag{29}$$

Table 1. Laufer's (1953) experimental data of $\overline{(\tilde{v}_{fs})^2}$ as a function of r/r_0

r/r_0	$\overline{(\tilde{v}_{fs})^2} \times 10^4$ (cm^2/s^2)
0.926	2.94
0.309	1.54
0.000	1.35

Table 2. Laufer's (1953) experimental data of t^* and ϵ_f, as a function of r/r_0

r/r_0	$t^* \times 10^{-4}$ (s)	ϵ_f (cm^2/s^2)
0.926	1.45	4.28
0.309	3.06	4.71
0.000	3.55	4.79

where T is the time period, $T = 2\pi/\omega$. The quantity, $\overline{(\tilde{v}_{fs})^2}$, which is nothing more than the square of the r.m.s. value of $\tilde{v}_{fs}$, is often measured experimentally and is commonly found in the literature as a log–log plot of the Eulerian energy spectrum as a function of frequency, $n = \omega/2\pi$ (see figure 4). From example, values of $\overline{(\tilde{v}_{fs})^2}$ have been extracted from experimental data Laufer (1953) has collected, mostly through hot-wire anemometry, in fully developed turbulent pipe flow having a Reynolds number of 500,000 and an axial velocity of 30.5 m/s (100 ft/s), and are summarized in table 1. The quantity r_0 is the inner radius of the pipe Laufer used.

Hinze (1959) states that for short diffusion times ϵ_f and ϵ_p can be evaluated as

$$\epsilon_f = t^* \int_0^\infty E_f(n)\,dn = \overline{(\tilde{v}_{fs})^2}\, t^*$$

and

$$\epsilon_p = t^* \int_0^\infty \eta^2 E_f(n)\,dn = \overline{(\tilde{v}_{ps})^2}\, t^*. \qquad [30]$$

It will be assumed that the characteristic diffusion time, t^*, used to evaluate the fluid and particle diffusivities in the above equations, is the time associated with one period of oscillation for the frequency, ω_e, associated with the average energy across the spectrum so that $t^* = 2\pi/\omega_e$. Therefore, once a definition for ω_e has been established, since $\overline{(\tilde{v}_{fs})^2}$ has already been experimentally determined, an expression for ϵ_f that can be obtained solely from experimental results, will be possible. The frequency, ω_e, associated with average energy across the spectrum can be determined from the information experimentally provided in a typical energy spectrum curve, as shown in figure 4. The area under such a curve has already been shown to equal $\overline{(\tilde{v}_{fs})^2}$. A horizontal constant energy spectral line can be determined such that the rectangular area under this curve equals $\overline{(\tilde{v}_{fs})^2}$. The frequency, n_e, at the point where the two spectral lines intersect corresponds to the average energy across the spectrum and equals $\omega_e/2\pi$ or simply $(t^*)^{-1}$. Values of the characteristic diffusion time, t^*, and hence the fluid turbulent diffusivity, ϵ_f, were determined from the experimental results provided by Laufer (1953), and summarized in table 2.

An expression for particle turbulent diffusivity, ϵ_p, as established in [30] may be evaluated experimentally as well. The amplitude ratio, η, is taken to be the amplitude ratio η_e corresponding to ω_e as defined by [23]. Since η_e has just been defined using the frequency, ω_e, associated with the average energy across the spectrum, it may be treated as an experimentally known constant in [30]. Thus the particle diffusivity can be written as

$$\epsilon_p = \eta_e^2 t^* \int_0^\infty E_f(n)\,dn = \eta_e^2 \epsilon_f, \qquad [31]$$

where the amplitude ratio, η_e, diffusion time, t^*, and integrated energy spectrum can all be determined experimentally.

Looking at the time-mean calculations previously undertaken gives rise to a physical description of the quantity

$$\int_0^\infty (\xi + \lambda)\,d\omega$$

found in [28]. From the definition of $\tilde{v}_{fs}$, [10], we have

$$|\tilde{v}_{fs}| = \int_0^\infty |(\xi \cos \omega t + \lambda \sin \omega t)|\,d\omega \qquad [32]$$

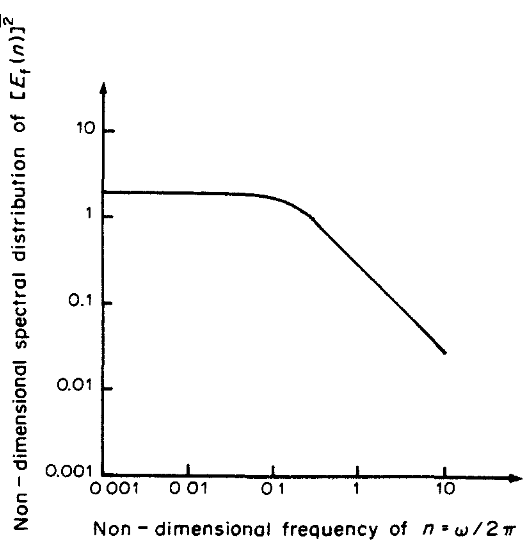

Figure 5. Typical spectral distribution of $[E_f(n)]^{\frac{1}{2}}$.

and

$$\overline{|\tilde{v}_{fs}|} = \int_0^{\frac{2\pi}{\omega}} \int_0^{\infty} \frac{|(\xi \cos \omega t + \lambda \sin \omega t)|}{\left(\frac{2\pi}{\omega}\right)} \mathrm{d}\omega \, \mathrm{d}t. \qquad [33]$$

When the amplitude functions ξ and λ are of the same order of magnitude, we have:

$$\overline{|\tilde{v}_{fs}|} = \frac{1}{\pi} \int_0^{\infty} (\xi + \lambda) \, \mathrm{d}\omega. \qquad [34]$$

A closer look at figure 4 reveals how the denominator in [28],

$$\int_0^{\infty} (\xi + \lambda) \, \mathrm{d}\omega$$

which now equals $\pi \overline{|\tilde{v}_{fs}|}$, may be determined experimentally. The area under the $E_f(n)$ curve for a particular unit width of frequency can be defined as $(\tilde{v}_{fs})^2$. The summation of these terms across all frequencies yields $\overline{(\tilde{v}_{fs})^2}$. Therefore, if the square root of the data plotted in figure 4 is replotted, as in figure 5, the area under the new curve for a particular unit width of frequency can be defined as $|\tilde{v}_{fs}|$. (Note that only positive values of the square root were taken.) Thus, the summation of these new terms across all frequencies yields $\overline{|\tilde{v}_{fs}|}$. This calculation has been performed on Laufer's (1953) data, where the results have been summarized in table 3.

It should now be apparent that a value of k can be obtained experimentally. From [28], [30] and [34], we have

$$k = \frac{-2\eta_e^2 \epsilon_f}{\int_0^{\infty} (\xi + \lambda) \, \mathrm{d}\omega} \frac{1}{\alpha} \frac{\partial \alpha}{\partial y} = \frac{-4}{\pi} \frac{\eta_e^2}{\omega_e} \frac{\overline{(\tilde{v}_{fs})^2}}{\overline{|\tilde{v}_{fs}|}} \frac{1}{\alpha} \frac{\partial \alpha}{\partial y}. \qquad [35]$$

Table 3. Laufer's (1953) experimental data of $\overline{|\tilde{v}_{fs}|}$ as a function of r/r_0

r/r_0	$\overline{|\tilde{v}_{fs}|} \times 10^2$ (cm/s)
0.926	6.8
0.309	3.4
0.000	2.6

Thus the result for the particle drifting or diffusion velocity, $\bar{\bar{v}}_{pd}$, can be written from [24] as

$$\bar{\bar{v}}_{pd} = \bar{\bar{v}}_{fd} = -\frac{2\pi}{\omega_e}\eta_e^2\overline{(\tilde{v}_{fs})^2}\frac{1}{\alpha}\frac{\partial\alpha}{\partial y}. \qquad [36]$$

SUMMARY AND CONCLUSIONS

This theoretical investigation has developed a model to explain transverse particle transport in turbulent flow due to turbulent diffusion. The model is based on the particles' ability, or inability, to respond to surrounding local fluid motion. Transverse particle motion has been divided up into two components, each separately driven by a particular type of fluid motion.

The first fluid–particle velocity pair investigated were Fourier integral representations of fluid and particle fluctuating velocities. It was shown that the particles' response in general lagged the driving fluctuating fluid velocity by a phase angle β, and had an amplitude modification of η. This type of fluid–particle response dominates regions where the fluid fluctuations have relatively large amplitudes, as compared to particle size.

The second fluid–particle velocity pair examined were local drifting velocities due to the effect on the oscillatory component of fluid motion by the concentration distribution of particles in the transverse direction. The local drifting velocity of the particle was found to be a function of the turbulence properties of the surrounding fluid and the fractional particle concentration gradient in the transverse direction. The local mean particle drifting velocity can be superimposed on to the particle fluctuating velocity to give a total picture of the dominating particle velocities found in regions characterized by large amplitude fluid fluctuations.

A subsequent paper (Lee 1987) will show that the theory (together with other new results on the concentration effects on particle drag and lift and fluid turbulence properties) can help to explain the phenomena measured by Lee & Durst (1982).

NOMENCLATURE

$a = \dfrac{36\,\mu_f}{(2\rho_p + \rho_f)d_p^2}$ = A constant, as defined in [5a]

$b = \dfrac{3\rho_f}{(2\rho_p + \rho_f)}$ = A constant, as defined in [5b]

$c = \left[\dfrac{18}{d_p(2\rho_p + \rho_f)}\right]\left(\dfrac{\rho_f\mu_f}{\pi}\right)$ = A constant, as defined in [5c]

d_p = Particle diameter

$E_f(n)$ = Lagrangian energy spectrum as a function of frequency

f_1 = A function of ω, as defined in [20a]

f_2 = A function of ω, as defined in [20b]

k = A function, as defined in [11]

k' = A constant of proportionality, as defined in [27]

$n = \omega/2\pi$, Frequency

$n_e = \omega_e/2\pi$, Frequency associated with the average energy across the spectrum

r_0 = Inner radius of the pipe

r_c = Cutoff radius

t = Time

t^* = Characteristic diffusion time

$T = 2\pi/\omega$ = Time period

$\tilde{u}_{fs}$ = Sinusoidal fluid motion, represented as a Fourier integral in the axial direction

$\tilde{u}_{ps}$ = Sinusoidal particle motion, represented as a Fourier integral in the axial direction

$\bar{\tilde{u}}_{ps}$ = Time-mean average of $\tilde{u}_{ps}$

$\mathbf{V}_f$ = Fluid velocity
V_f = Magnitude of fluid velocity
$\mathbf{V}_p$ = Particle velocity
$v_f = \tilde{v}_{fs} + \bar{\tilde{v}}_{fd}$ = Fluid velocity in the transverse direction
$\tilde{v}_{fs}$ = Sinusoidal fluid motion, represented as a transverse Fourier integral in the transverse direction
$\tilde{v}_{fd}$ = Additional local transverse fluid motion due to fractional variation in the particle concentration
$\bar{\tilde{v}}_{fd}$ = Time-mean average of $\tilde{v}_{fd}$, the apparent fluid drifting velocity in the transverse direction
$v_p = \tilde{v}_{ps} + \bar{\tilde{v}}_{pd}$ = Particle velocity in the transverse direction
$\tilde{v}_{ps}$ = Sinusoidal particle motion, represented as a Fourier integral in the transverse direction
$\bar{\tilde{v}}_{pd}$ = Additional local transverse particle drifting or diffusion motion due to fractional variation in the particle concentration
x = Axial coordinate
y = Transverse coordinate
α = Time-mean particle volumetric concentration
β = Phase angle, as defined in [19]
ϵ_f = Fluid turbulent diffusivity
ϵ_p = Particle turbulent diffusivity
η = Ratio of particle oscillation amplitude/surrounding eddy oscillation, as defined in [18]
η_e = Value of η associated with ω_e
λ = Complex Fourier integral amplitude component of fluid oscillation, as defined in [7]
ξ = Complex Fourier integral amplitude component of fluid oscillation, as defined in [7]
μ_f = Intrinsic absolute fluid viscosity
$\nu_f = \mu_f/\rho_f$ = Intrinsic kinematic fluid viscosity
ρ_f = Intrinsic fluid density
ρ_p = Intrinsic particle density
$\omega = 2\pi n$ = Frequency
ω_c = Cutoff frequency
ω_e = Frequency associated with the average energy across the spectrum

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